Field testing and optimising efficiency of hydro turbines

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An outline is given of the advantages and techniques of conducting both absolute and relative efficiency tests of hydraulic turbines. It is suggested that organizing a team to carry out index testing throughout the USA could increase the nation's output considerably.

IT IS INVARIABLY in the best interests of a powerplant to have the efficiency of its hydraulic turbines measured at the start of operation and subsequently at regular intervals; there are several different test methods available. Usually, the performance of large turbines is determined first in model tests. However, prototype turbine installations always have some differences from their models which alter their performance characteristics.

In general, efficiency testing of the prototype is used to:

- check that contract guarantees have been met;
- adjust the blade-gate mechanism of Kaplan turbines for optimum efficiency;
- check for proper adjustment and operation of the governor;
- determine the optimum wicket gate position for full closure of the vacuum breaker on Francis turbines;
- obtain information that can be used to evaluate machine wear and cavitation degradation without having to dewater and inspect;
- obtain accurate information on water use.

However, no matter what prototype testing is done, turbine model tests are still needed to determine the full performance capabilities of a particular machine. The model must be subjected to conditions that would be detrimental to a prototype such as severe cavitation and maximum runaway speed. These conditions are obviously seldom simulated again in the prototype for testing purposes. In fact, while testing a large, high-head Kaplan to maximum on-cam runaway speed ten years ago, the author still remembers watching the entire concrete monolith bouncing and separating from the rest of the powerhouse.

Model efficiency curves are usually determined to a high degree of accuracy. However, there are inherent differences between a model and its test-stand instrumentation, and a prototype and its powerhouse features, that cause some differences between the efficiency curves of the two. These differences are caused by such factors as:

Efficiency stepup. There is a scaling effect caused by the size difference between the model and prototype of any kind of turbine. This usually results in both an efficiency stepup and a shift with respect to power of the prototype. A recent analysis has been done on comparing the point of peak efficiency between model and prototype Francis turbines[1].

The cause of this effect has been hypothesized by some as being the change in the boundary layer ratio. In general, the higher the Reynolds number, the higher the velocities and the thinner the laminar boundary layer. A prototype invariably operates at a Reynolds number

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Notations

A =	Cross-sectional area of flow passage (ft^2)
D =	Differential in pressure between two piezometers (ft of water)
E =	Efficiency
g =	Gravitational acceleration (ft/s)
H =	Head (ft)
HP =	Turbine output (h.p.)
k =	Piezometer differential constant ($ft^{5/2}/s$)
kW =	Generator output (kW)
ln =	Naperian logarithm (base e)
P =	Pressure (lb/in^2) (gauge)
Q =	Discharge (ft^3/s)
r =	Radial distance from centre of rotation (ft)
γ =	Density of water (lb/ft^3)
c =	Subscript denoting a common value to which individual test values are converted
t =	Subscript denoting individual test point values

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which is several orders of magnitude higher than that of its model. Consequently, the equivalent boundary layer in the prototype is thinner than in the model which allows an increase in the effective flow passage areas.

Powerhouse head determination. Model tests in the USA are usually conducted according to either the ASME[2] or IEC[3] test codes. These define the net head on a turbine as the difference in energy levels between the fluid entering and leaving the turbine. This means that at the point of measurement, the velocity head is added to the water surface elevation or piezometric head to obtain the total energy level. However, the instrumentation in most powerhouses simply monitors the elevation of the water surfaces, or the piezometric heads, without any velocity head correction. Thus, if monitoring takes place in areas where there are significant velocity heads (such as in penstocks and tailraces) head measurements may be different from those on the model. (The essential difference between the two test codes is that the ASME code charges the sudden expansion losses of the draft tube to the turbine while the IEC code does not.) Another difference in determining head on a model and prototype is caused by hydraulic losses of the trashracks. Because of the scaled down size of the equivalent grating, these cannot be simulated physically in the model test. Therefore, in those prototype installations where the upstream energy level is the forebay elevation, the reduction in head on the prototype turbine caused by trashrack losses would not be duplicated in the model.

Site differences. The coefficient of gravitational acceleration at a prototype powerhouse is different from that at a model test laboratory because of the differences (in latitude and altitude) of the two places. Also the water temperature may vary, and laboratory water is usually deaerated and treated with anticorrosive additives. The end result is a difference in the density of water between a prototype powerhouse and a model test laboratory. The denser the water, the more potential energy it contains for a given difference in water surface elevations. Expressed as an example in a different manner, a 199 ft (60.6 m) column of pure water at 40°F (4°C) exerts the same pressure at the same location as a 200 ft (61 m) column of 89°F (32°C) water.

Manufacturing differences. Even with the best quality and dimensional controls during manufacture, there are always some differences between the prototype turbine and the theoretical equivalent model. For instance, most manufacturers cast their runners with the vent areas (minimum cross-sectional flow passage areas) several per cent larger than the equivalent model. This is to accommodate anticipated swellage of the casting and to ensure any error in fabricating the runners is on the side of more, not less, power.

Deflection differences. The magnitude of loads relative to component rigidity is different in the prototype compared with the model. This combined with materials which have different mechanical properties, used in making the model and the prototype, results in different equivalent component deflections. The basic parameter for similitude of turbine test data is the peripheral speed coefficient. This relates the speed of the periphery of the runner to the available, or spouting, velocity of the fluid. It contains the term for rotational speed and runner diameter in the numerator and a constant and the square root of head in the denominator.

Models, however, are usually tested at higher speeds and heads than the equivalent prototype, but such that the ratio of speed and diameter to the square root of head duplicates the peripheral speed coefficient of the prototype. The higher model speed, however, causes a greater centrifugal force on the blades of a Kaplan or propellor model than on the prototype.

The resulting force vector changes the amount of downward deflection that the hydraulic loading would normally cause. However, because of the different equivalent heads the hydraulic loading is also different between a model and a prototype.

To account for these effects, some prototype Kaplan units are made with a vertical offset between the inner and outer blade trunnion bushings. Thus, the prototype blade has a slight upward dihedral when unloaded compared with the loaded condition. Further, a turbine model cannot simulate the deflection under load of the generator thrust bearing which takes the entire hydraulic thrust as well as the weight of the rotating parts. To counter this, some prototype Francis turbines are made with a vertical offset between the vertical centreline of the runner and the centreline of the distributor. In addition, the shaft, particularly of a prototype Kaplan, which is hollow to allow for the Kaplan operating rod and oil pipes, elongates under hydraulic load.

The blade angle sensing mechanism is usually indicated off the top of the operating rod or one of these oil pipes. Consequently, its elongation with the shaft causes a different blade angle indication compared with the blade angle calibration, which must necessarily be done in the dry.

Wear. Distortion of the measurements on the prototype may occur as a result of the aging process in which residual internal material stresses are released, even after the normal stress relieving performed after casting and welding. Also most turbines suffer some cavitation and erosion damage. This is usually repaired periodically by welding. Such welding, no matter how expertly done, results in some changes to the surface contour and surface finish by weld material buildup and by thermal distortion. Thus, even prototype units with good maintenance procedures are subjected to changes of contour over their operating life.

Types of efficiency testing

There are two basic types of turbine efficiency testing—the absolute and the relative. These terms denote whether the discharge is measured absolutely or in relation to some other known parameter. The absolute methods are invariably more complex, expensive, and difficult; and are generally done only once to ensure the contract guarantees have been met. These methods include:

- Gibson [2,4,5]—the change in momentum of the fluid column resulting from a load change is measured to determine the flow rate.
- Allen Salt Velocity[2,5]—the time for the passage of an injected saline solution between two electrode stations is measured to determine the flow rate.
- Weir[2,5]—a calibrated weir, usually downstream, is used to measure discharge.
- Piezometer Traverse[2,5]—piezometers are used to determine the velocity profile which is integrated over the cross-sectional area to determine flow rate.
- Current meters[2,5]—same as the piezometer traverse except for the use of current meters instead of piezometers.
- Sonic—acoustic transducers are used to measure the doppler or phase shift between two stations the average flow rate separating the two.
- Thermodynamic[5]—minute changes in the temperature of the discharge water compared with the incoming are used to evaluate efficiency.

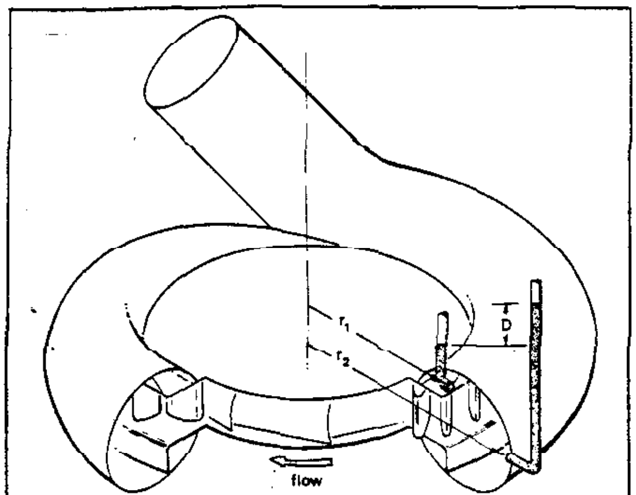

Fig. 1. Typical Winter-Kennedy piezometer installation.

Relative efficiency testing, on the other hand, is considerably easier, for the flow is only measured relative to, or is indexed against, some parameter which is a direct function of flow rate. Almost any parameter will do, such as a venturi section in the penstock or the frictional head loss in a penstock or the centrifugal force of the flow around a bend, and so on. The latter tends to be most commonly used, since a venturi section in a penstock causes permanent additional head loss and frictional penstock losses tend to change with time. In essence, index testing allows the performance of a unit to be determined over its full operating range more easily than any other method.

The best known indexing method is the Winter-Kennedy[6] piezometer system depicted in Fig. 1. This system includes two piezometers in the initial curved portion of a spiral or semi-spiral case. The elevation of either individual piezometer tap in relation to the other is of no consequence, however, they must be on the same radial line. Since the piezometer differential is a function of the velocity head, $D \alpha V^2/2g$, and since by continuity, $V=Q/A$, the differential is proportional to Q^2 or conversely, $Q \alpha \surd D$. In fact, for the specific derivation of this relationship, the radial pressure gradient of a free vortex in the horizontal plane is given[7] by:

$$\mathrm{d}D/\mathrm{d}r = V^2/gr$$

Integrating between the two piezometers:

$$D = \int_{r_1}^{r_2} V^2\, \mathrm{d}r/gr = V^2 [\ln r_2 - \ln r_1]/g = V^2 \left[\ln (r_2/r_1)\right]/g$$

Substituting again from continuity:

$$D = Q^2 \left[\ln (r_2/r_1)\right]/A^2 g$$

or again conversely:

$$Q = \surd D\ (A) \surd\{g/[\ln (r_2/r_1)]\}$$

In those absolute efficiency tests where the particular Winter-Kennedy piezometers have been calibrated simultaneously, the square root-linear relation has been found to be extremely accurate and applicable over the entire flow range.

Index testing is a straightforward procedure. There are, however, pitfalls that need to be avoided. Considering the various types of turbines, Francis and fixed-blade propellor turbines are tested in identical manners and are considerably easier to test than Kaplans. In fact, three or four people can test a unit in a couple of hours. The basic method is to measure the Winter-Kennedy differential, generator power output, wicket gate servomoter stroke, and head. The Winter-Kennedy's are most easily measured by a differential water manometer which requires the reading of only one water surface elevation. For those relatively rare piezometer systems which intrinsically have little total pressure differential, the manometer may be inclined at a known angle to amplify the distance the water surface moves and then the equivalent vertical elevations computed.

Generator output is usually measured by timing the switchboard rotating watt-hour meter. Wicket gate servomotor stroke is most conveniently measured by a machinist's scale and pointer, attached to the servomotor piston rod. On the scale there are two initial or starting points from which to select. One corresponds to a calibrated zero gate opening and the other to a full governor pressure squeeze on the gate mechanism. The difficulty with the former is the hydraulic loading in the watered condition, which, being a function of head, tends to change the amount of servomotor stroke at which zero gate occurs compared with the unwatered calibration. Therefore, the preferred starting point is that which is achieved consistently by full squeeze on the gates.

However, when comparing for equivalent gate openings with model data, which is on an actual gate opening basis, the necessary conversion for the length of squeeze stroke should be made.

There are various definitions of head such as net and gross, and as noted earlier, the test codes themselves even define net heads differently. However, for index testing the head should be determined in exactly the same way as the powerhouse instrumentation evaluates it, whatever the definition. This emphasizes the essential difference between absolute and relative testing. It is not simply whether the discharge is measured absolutely or not, but that absolute testing is generally used to evaluate the performance of a turbine against a set of criteria such as contract guarantees.

Relative testing, on the other hand, is to determine the relative performance of the entire turbine-generator unit as controlled by its governor operating with the specific powerhouse data inputs. In other words, it is a complete on-site system evaluation as opposed to individual component evaluation.

To index either a Francis or a fixed-blade propellor turbine, all that is needed is to open the wicket gates to a given point, block them, and let operating conditions such as head, generator temperature, and so on, stabilize. Since wicket gates are usually designed hydraulically to have a closing tendency, moving in the opening direction provides the resistance to account for any looseness in the gate linkages.

As soon as conditions stabilize, the four quantities are recorded and the gates are opened to the next point. To reduce these data, two basic steps are needed.

First, since some head variation may occur in the time between the different gate positions, the data must be corrected to a common head. For small head variations the affinity laws dictate that the power should be multiplied by the head ratio to the power of 3/2, while the square root of the piezometer differential should be multiplied by the square root of the head ratio, that is:

$$\mathrm{k}W_c = kW_t\,(H_c/H_t)^{3/2}$$

$$\surd D_c = \surd\left[\mathrm{D}_t\,(H_c/H_t)\right]$$

For larger variations the same proportional change

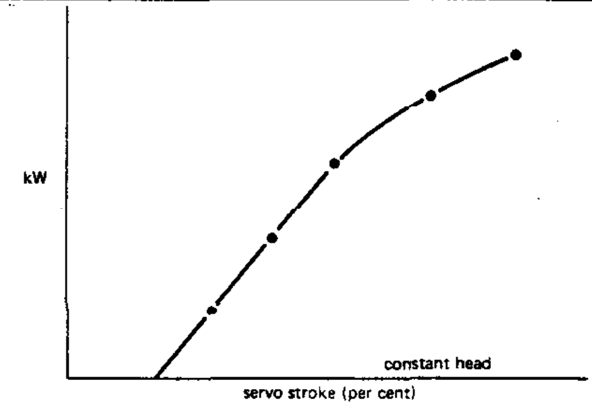

Fig. 2. Power output versus servo-stroke at constant head.

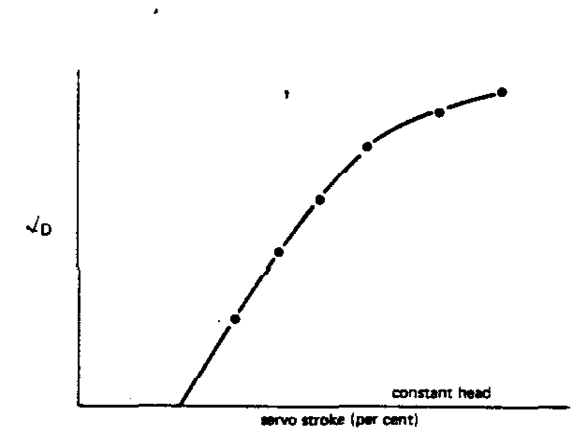

Fig. 3. Piezometer deflection versus servo-stroke at constant head.

determined from the model performance curves must be used.

Second, the relative efficiency curve is determined. This could be done directly by computing and plotting the efficiency points and drawing a best fit curve. However, efficiency curves frequently have irregular contours, so that it is difficult to make an accurate graph of them. There is, however, a constructed curve technique to eliminate this problem. If the power and piezometer differential data are each plotted against servo-stroke, as shown in Figs. 2 and 3, the data points inevitably define smooth, continuous, nearly linear, curves (with one slight but well defined "knee"). A best-fit curve can be constructed accurately through such uniform data points. Then interrogating each curve at the same servo-stroke, a constructed curve value of $kW/\sqrt{D}$ is computed. This value is recognised by comparison with the basic power formula $E = 550\,HP/Q\gamma H$ as being a relative efficiency of the combined turbine-generator system for a given head and density of water.

These constructed curves of relative efficiency and percentage servo-stroke may be plotted against power or piezometer deflection as shown in Fig. 4. Finally, the individual test point values of relative efficiency are computed and plotted for comparison with the constructed curve to check for excessive deviation of any individual test points. This index test procedure may be repeated for whatever other heads are needed to define the full operating range of the unit.

Although this test has been done in a relative manner, there is now a way of presenting these data in terms of actual discharge, even if the actual prototype discharge has never been measured. For this, three things need to be known: the density of water during the index test; the peak turbine model efficiency at the index test head; and, the generator efficiency at the point of peak prototype relative efficiency. The density of water during the index test may be measured directly or computed from standard references if the water temperature is measured. The direct measurement may be obtained whenever the unit is shut down, but watered to the forebay. It is just necessary to measure accurately the pressure on any piezometer upstream of the wicket gates. Generally a dead-weight tester is used for this measurement. With this pressure, and knowing the elevation difference between the forebay and the dead-weight tester, the water density is found from $\gamma = 144\,P/H$. From the turbine model data, all that is needed is the value of peak efficiency at the equivalent head (and method of determining head) at which the prototype index test was conducted. This model peak efficiency may be stepped up by appropriate scaling laws[1,7]. The location of the model peak efficiency in relation to the model power, gate opening, and so on, is not needed. Thus, any power shift in the point of peak efficiency between model and prototype will not affect this computational procedure. Finally, if the generator efficiency is known at the prototype power at which the relative peak efficiency* occurs then the actual turbine horsepower output at that point may be computed. Then since horsepower, head, efficiency, and water density are all known at the same point, the prototype discharge may be computed at that point from the above basic power formula. For the final step it is noted that the value of $\sqrt{D}$ is also known at this point of peak prototype relative efficiency. Thus, since the discharge, Q, is directly proportional to $\sqrt{D}$, the Winter-Kennedy piezometer calibration may be determined from $k = Q/\sqrt{D}$. As a result of this procedure, a piezometer calibration constant applicable over the full flow range and independent of head may be obtained which can be used to convert all of the relative performance data of Fig. 4 into actual performance data. This may be in terms of actual discharge and actual turbine or turbine-generator power output and efficiency.

The only untested value to induce an error is a

*This peak represents a combination of the generator and turbine efficiency curves. The procedure is to convert the relative efficiency curve from terms of generator output to turbine output and to determine that peak. However, since generator losses are of the order of one-tenth as much as turbine losses, the difference in these two peaks is usually insignificant.

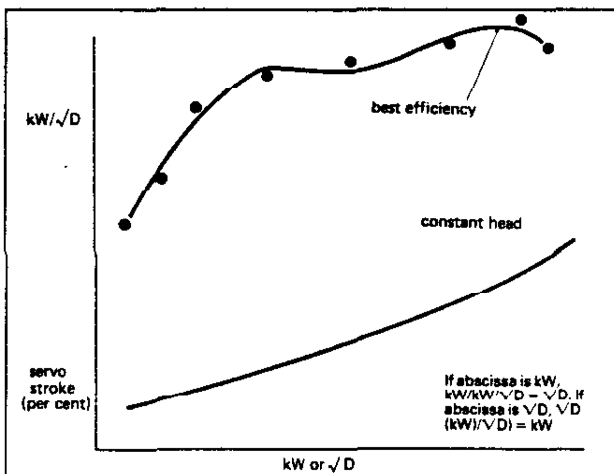

Fig. 4. Constructed relative efficiency curve with individual relative efficiency data points (above) and corresponding smoothed curve (below).

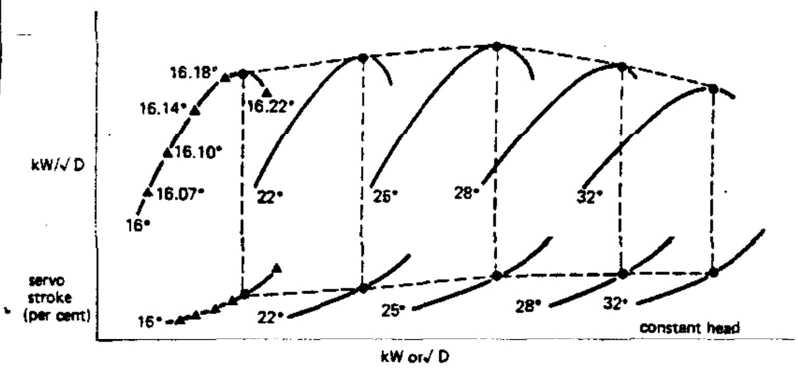

Fig. 5. Relative efficiency and smooth curves for fixed blade positions (solid line) with tangent curve for optimum Kaplan efficiency (dotted).

difference, if any, between the peak prototype and the stepped up peak model efficiencies. If absolute efficiency testing has been done, the Winter-Kennedy piezometers can be calibrated without even this possible error.

The results from index testing may be used in several ways:

- A calibrated gauge may now be attached to the Winter-Kennedy piezometers to read actual flow rate accurately, as well as recording instrumentation, to determine the total discharge over a set period of time. The latter has, in the past, been referred to as a water register.
- The machines can now be operated at their peak efficiency to generate the most energy from the available water.
- The performance can be checked and any wear or degradation of the machinery can be evaluated at any subsequent time without dewatering, by noting the head, and reading the Winter-Kennedy gauge and power output to determine efficiency, and comparing with the previous tested performance. When comparing any constant head relative efficiency curves where there are significant variations in water density, generally as a result of large temperature differences, the relative efficiency curves can be converted to a common basis by:

$$kW_c/\sqrt{D_c} = (kW_t/\sqrt{D_t})\,(\gamma_c/\gamma_t).$$

- If the units are equipped with vacuum breakers to reduce vibration during part-gate operation, an index test with the adjustable air valve fully open and then fully closed will allow setting of the gate opening by which the valve is to be fully closed for most efficient operation.
- If absolute testing has been done at any head, the much easier index testing can be used to determine the absolute performance at any other head.

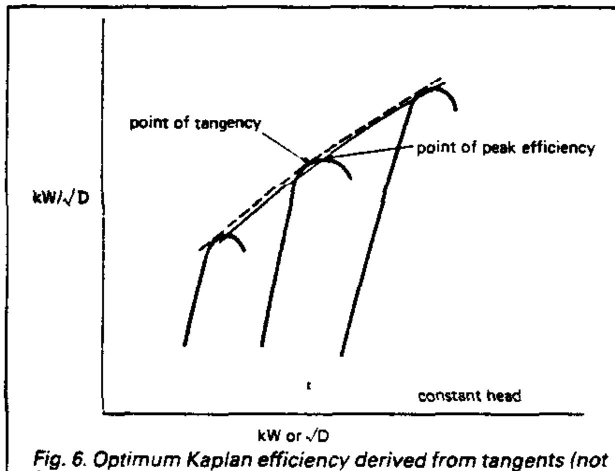

Fig. 6. Optimum Kaplan efficiency derived from tangents (not from individual points of peak efficiency).

In addition to the foregoing, the pre-eminent advantage of index testing is to determine the optimum blade-to-gate relationship of Kaplan turbines so they can operate at their best efficiency. In conventional Kaplan turbines, which are adjustable pitch propellor turbines, the pitch of the blades follows the opening of the wicket gates. That is, at a constant head, there is for any gate position a unique blade angle which allows the most efficient operation of the turbine. Thus, the efficiency of a Kaplan machine is extended over a wide load range, compared with a fixed blade propellor unit. The position of the blades is usually controlled by a governor acting in response to a mechanical cam curve. There is usually a series of such cams to cover the full operating head range. These have generally been determined by model tests. However, because of the previously mentioned differences between a prototype installation and a model test, the optimum cam curves for the prototype invariable have some differences from the cam curves derived from model tests. Some newer hydro installations use electronic circuits in their governors instead of mechanical cams to adjust the runner blade angles. However, the input-output parameters of the circuits need, of course, to be adjusted just as the mechanical cams do.

The method of indexing a Kaplan turbine is the same as described previously, but with some extra characteristics. The additional parameter of blade angle is handled literally by holding it constant. That means the blades are blocked in a fixed position, then the gates are opened in gradually increasing stages, and the readings taken at each increment. Then the blades are moved to a new position and the process is repeated.

Using the constructed curve technique, the relative efficiency curves for each fixed blade angle are plotted as shown in the upper part of Fig. 5. Next, the tangent curve to these blade angles, shown as a dashed line, is constructed.

It is important to note that this curve should be the tangent to the fixed blade angles and does not, except in the case of the unique blade angle of peak efficiency, actually pass through each individual fixed blade point of peak efficiency. This anomaly is presented graphically in Fig. 6 where the abscissa has been compressed to show the reduction in efficiency which occurs over the power range if any curve other than the actual tangent curve is used.

Consequently, obtaining an accurate tangent curve in graph form depends on the accuracy with which the fixed blade efficiency curves are drawn, which in turn shows the advantage and necessity of using the constructed curve technique.

The next step is to make a graph of the corresponding servomotor stroke curves as shown in the lower portion of Fig. 5. Then the points of tangency from the above fixed

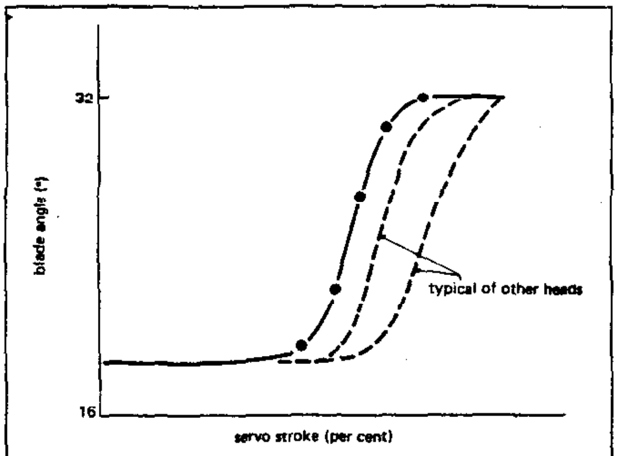

Fig. 7. Kaplan cam curves.

blade efficiency curves are projected directly down to these servomotor curves. These intersections now denote the servo-stroke position where a given fixed blade angle produces power most efficiently.

If these intersections are replotted in the form shown in Fig. 7, they conversely show the blade angle needed by a given servo-stroke for optimum efficiency, and in fact become the required cam curves.

There is one final data reduction technique to be developed. That is the specific blade angle at the point of tangency. It is noted, incidentally, that there is no universal convention for defining blade angles; each manufacturer has a different one. Usually, it is based on the angle formed by two arbitrarily defined points on the upper blade surface with the horizontal plane. However, some manufacturers use the straight line between these points as the hypotenuse while others use the actual distance on the blade surface and others use the "extended cord" based on the distance around the outer blade periphery.

As previously mentioned, even if the blades are blocked in a fixed position, the hydraulic thrust causes a shaft elongation which in turn causes an apparent change in blade position to the indicating mechanism of the governor.

The standard method to attempt to compensate for this is to measure the indicated difference in blade position between a hydraulically loaded and unloaded condition for an actual fixed blade angle. The blade calibration curve that has previously been obtained in the unwatered condition is then shifted uniformly by this difference.

The usual technique to measure this difference is to put the blades in a completely flat position, with maximum governor pressure in an unloaded condition, and record the indicated blade position. The preferred unloaded condition is, of course, dewatered.

However, this indicated blade angle is usually recorded in the watered-with wicket gates closed, or the speed-no-load condition, as these are usually more convenient. The blades are kept completely flat under full pressure and the gates are then opened to where the blades would normally start to move, and this indicated blade position is compared with that previously recorded to find the difference. However, there is a more accurate data reduction technique. That is to record the indicated blade position based on the unwatered calibration at each data point as shown for the nominal 16° blade angle in Fig. 5. Then, by interpolation either along the length of the fixed blade efficiency curve or by the abscissa units as shown in Fig. 8, the apparent blade angle at the point of tangency is determined. This again emphasizes the difference between absolute and relative testing. In relative testing, knowledge of the actual blade position is not important. The critical item is where the governor detects that the blades are.

The benefits that can accrue from index testing may be seen conversely by noting what happens when it is not done. The North Pacific Division (NPD) is the largest hydropower division in the US Army Corps of Engineers. At present they operate 105 Kaplan turbines with a total rated capacity of 9 757 470 kW. Most of these units have never been indexed, and whether they are operating at or near their optimum efficiency is unknown. The author was, however, able to index three of these, selected on a random availability basis, several years ago.

The index data for the machines as found was compared with the index test results (the relative efficiencies at various gate positions can be initially measured even before the governor cabinet is opened and these later plotted on the results of the index test). This comparison showed that these three turbines were operating at less than their optimum capabilities.

One of the three was operated by a governor with the newer 3-D cam. This is a cam that is continuous both in blade-angle-to-gate and in head. Thus, the powerplant operators do not need to change the individual cam plates when head changes to maintain optimum performance.

The test results showed, however, that the cam was not responding to a change in head. These three indexed units have never subsequently been adjusted to optimize their performance.

If these three test results are typical, and given the magnitude of the total NPD installed hydro capacity, it is likely that a considerable amount of additional energy could be generated by optimizing the performance of these units.

Extending these observations to the rest of the hydro generating plants in the Pacific Northwest, and even to the rest of the nation, it is the author's opinion that substantial amount of additional energy could be generated by a national program to optimize the performance of all existing hydro generating equipment. Also, it is evident that as personnel become familiar with this type of testing, the speed and accuracy of individual tests will also increase. Consequently, it would be financially beneficial for a government or utility agency to establish and make available a specific team of test personnel who would conduct highly accurate index tests on request. This test team would have to be available when water conditions dictate. The plant owners, on the other hand, would be expected to operate their generating equipment for the test personnel who would only instrument, collect, record, analyze, and report the data.

Not only could such a team develop expedient test procedures, but they could also develop new methods. One such possibility is the innovative concept proposed by Beckwith Electric Corporation[8]. To illustrate this

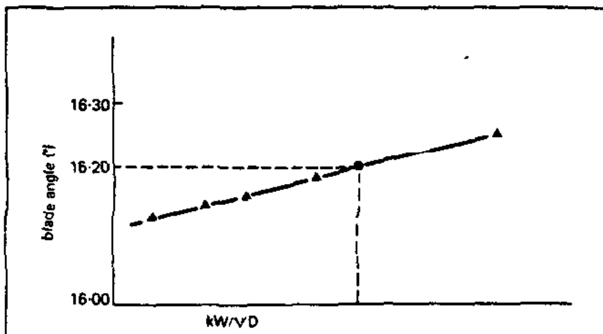

Fig. 8. Graphical interpolation for exact blade position at point of tangency.

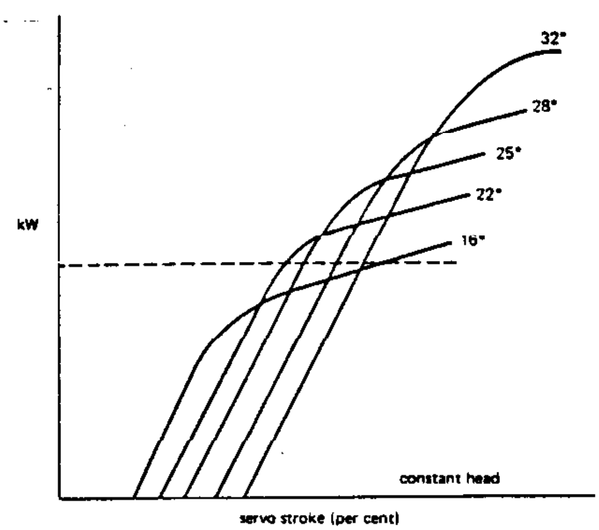

Fig. 9. Limits for vibrational displacement.

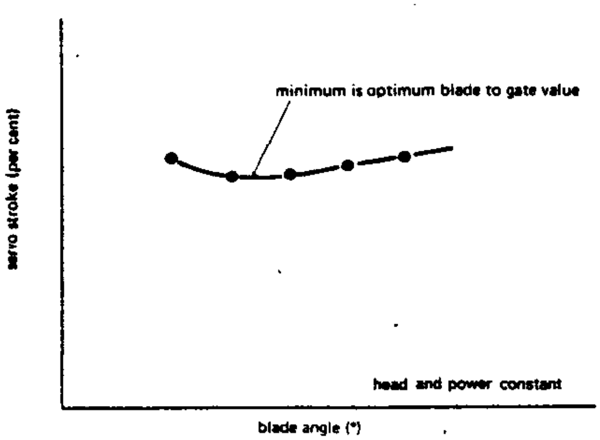

Fig. 10. Servo-stroke versus blade angle at constant head and power output.

concept which they developed from control theory, the graph of Fig. 2 is first expanded to include the other tested blade angles and then a horizontal line at constant power is drawn as shown in Fig. 9. A cross-plot of the points of interception is constructed as shown in Fig. 10, and the minimum point is the optimum blade to gate value. In other words, the blade angle which allows the smallest gate opening to produce a given constant amount of power is the optimum blade angle for that gate opening. Subsequently, as many constant power lines as desired may be constructed to define the full cam curve. This method eliminates the need to measure discharge even relatively. However, for most Kaplan turbines it turns out that the curve of Fig. 10 is so shallow, that finding an accurate minimum (given the accuracy of the mechanical response system) would be very difficult. □

Note

The author has advised us of a number of changes to his text which were too late for inclusion. For "piezometer" read "Pitot tube" and for "density" read "specific weight".—*Editor*

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Acknowledgement

The author would like to thank B. Eyden of Brown and Root Inc. for his review and comments on the draft of this paper.